Control Systems Laboratory (EE 3321) — Experiment 3

MATLAB Transfer Functions, Systems, and Control System Characteristics

I. Overview of Experimental Procedure

This experiment utilizes MATLAB to construct systems of transfer functions represented in the Laplace domain. During the experiment, the student will gain an understanding of the usefulness and visualization of the Laplace transform, the idea of system transfer functions, and some control system configurations and stability characteristics.

II. Laplace Transform Theory

The Laplace transform is a convenient tool in the analysis of linear time-invariant systems. In particular, we can transform the system set of integro-differential equations or the system representation (component or block diagram). This permits us to find algebraic transformed equations that represent the system. We use these algebraic equations to solve for the transform of the output signal *Y(s)* after we have computed the transform of the input signal *X(s)* and supplied the initial conditions. The algebraic solution is simpler that a differential equation solution. All that remains to find the output signal at this point is to compute the inverse transform of *Y(s)*. This can easily be done for rational transforms by using partial fraction expansion, though the computations can be tedious.

Under the condition of zero initial conditions, the differential operator, *d/dt*, can be represented in the Laplace domain by *s*; conversely, the integral operator is represented by *1/s* in the Laplace domain. As an example, consider the spring-mass-damper system shown in Figure 2.1.

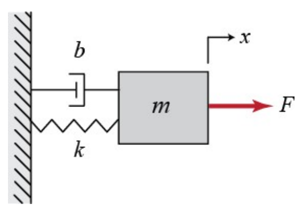

FIGURE 3.1: BASIC SPRING-MASS-DAMPER SYSTEM

This system can be described by the following integro-differential equation:

$$m\frac{d^2}{dt^2}x(t)+b\frac{d}{dt}x(t)+kx(t)=F(t) \quad (3.1)$$

Taking the Laplace transform of equation 2.1 under zero initial conditions yields

$$ms^2\,\mathrm{X}(\mathrm{s})+bsX(\mathrm{s})+kX(\mathrm{s})=F(\mathrm{s}) \quad (3.2)$$

which can be solved for X(s) algebraically.

III. Transfer Function Theory

The transfer function of a linear system is defined as the ratio of the Laplace transform of the output variable to the Laplace transform of the input variable, with all initial conditions assumed to be zero. Using the spring-mass-damper system as an example, the resulting transfer function is described by

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} \tag{3.3}$$

Additional transfer function models can be found in the lecture course textbook in *Table 2.5: Transfer Functions of Dynamic Elements and Networks*. This table includes Operational Amplifier circuit systems, motor systems, measurement systems, and other mechanical systems. The transfer function is extremely important to us since it completely characterizes the system except for initial conditions.

Creating a transfer function in MATLAB is accomplished by simply using the command `tf(num, den)`, where num is the polynomial of the numerator expressed in descending order (i.e., the polynomial $3x^2+5x+10$ is represented by [3 5 10]); den is the polynomial of the denominator.

To calculate the step response of a system, use the commands `[x, t] = step(sys, t)`, where t is the indexed time array and sys is the system described by a transfer function. You can calculate the impulse response in a similar fashion. Note that the impulse response of a transfer function will yield the time-domain signal represented by the Laplace transform function.

IV. Control System Characteristics Theory

We can use the function **roots** on the transfer function denominator polynomial to determine system pole locations. From these pole locations, we can determine the system stability characteristics. To plot the poles and zeros, use the command `pzplot(sys)`, where sys is the system described by a transfer function.

For a stable system, we can also find the frequency response directly from the transfer function by replacing s with $j\omega$. Therefore, the amplitude response is a slice through the transfer function amplitude surface along the imaginary axis. Also, the important system parameters of break frequencies and damping ratios can be found directly from the transfer function, since its coefficients are the same as those of the frequency response. This also means that we can plot the straight-line approximation to a system's Bode amplitude and phase responses directly from the transfer function. This straight-line approximation can be used to illustrate basic amplitude and phase response characteristics and help us choose any control system compensation that may be necessary.

V. Experimental Procedure

1. Consider the two polynomials $p(s) = s2 + 5s + 4$ and $q(s) = s + 10$. Compute the following:
   (a) roots of $p(s)q(s)$
   (b) poles and zeros of $G(s) = q(s) / p(s)$
2. **Plot** the pole-zero plot and the signal that corresponds to the transform
   $$X(s) = \frac{1}{s + c} \tag{3.4}$$
   for $c = 0.03$, $c = 0.3$, and $c = 3$. Note the effect of the pole location on the signal's characteristics.
3. A system has a transfer function
   $$\frac{X(s)}{R(s)} = \frac{(15/z)(s+z)}{s^2 + 3s + 15} \tag{3.5}$$
   Plot the response of the system when $R(s)$ is a unit step function for the parameter $z = 3, 6, and 12$.

4. Use Figure 2.1 to create a transfer function representing a simple spring-mass-damper system. Also consider the standard second-order transfer function equation

$$G(s) = \frac{\omega_n^{\;2}}{s^2 + 2\varsigma\omega_n s + \omega_n^{\;2}} \qquad (3.6)$$

For system parameters $m = 2$ *(kg)* and $b = 10$ *(N/(m/s))*, **determine** the spring constant value k that yields a critically damped response, or $\zeta = 1$. **Plot** the pole-zero plot for the value of k determined. Also **plot** for values of k an order of magnitude higher and lower than the determined value. Then **plot** the response signal for a unit impulse force and unit step force.

5. Use Figure 2.2 to create a transfer function representing an armature-controlled DC motor system.

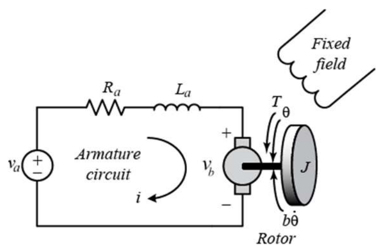

FIGURE 3.2: ARMATURE-CONTROLLED DC MOTOR SYSTEM

Use the given system parameters of $J = 0.2$ *(kg m²)*, $b = 10$ *(N m/(rad/s))*, $Ka = Kb = 0.45$, $R_a = 6$ *(Ω)*, and $L_a = 10$ *(mH)* to **determine** the voltage-speed transfer function. **Plot** the response of the system when it is subjected to an armature voltage step from 2 (V) to 3 (V) at $t = 0$.

VI. Conclusion

1. What conclusions have you drawn regarding the pole locations and system response? Be specific in your answer, mentioning the significance of the real- and imaginary-axis and importance of negative real poles and zeros.